Case Study: Deep Hedging

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Introduction
Overview

- Problem setting.
- Black Scholes Model and Heston Model.
- Delta hedge for a European Call Option.
- Deep reinforcement learning for hedging.
- Training of the neural network.
- Implemented algorithms and conclusion.

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Introduction
Problem setting

- Risk management and pricing under idealized complete markets.
- Hedging based on greeks from certain stochastic models.
- Is Machine Learning fixing the short comings of underlying models?
- Main paper of interest: *Deep Hedging: Hedging Derivatives Under Generic Market Frictions Using Reinforcement Learning. (March 2019). Swiss Finance Institute Research Paper*.

Black Scholes Model
Model & Assumptions

- Assumptions:
  - → The stock price follows a Geometric Brownian Motion.
  - → Interest rate and volatility of stock assumed to be constant.
  - → No dividends, no transaction costs.
- Risky asset dynamics:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

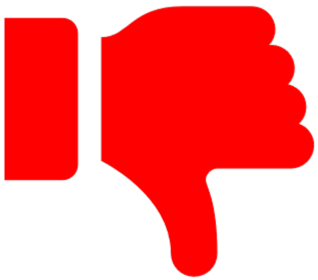
- Formula for a European Call Option:

$$C(S_t, t) = N(d_1) \cdot S_t - N(d_2) \cdot K \cdot e^{-r(T-t)}$$

from BS equation $\dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + rS\dfrac{\partial V}{\partial S} - rV = 0.$

Black Scholes Model
Short-comings

- Risk-free rate and volatility are dynamic in reality.
- Jumps in prices are not taken into account.
- Log returns are not necessarily normal distributed.
- The series of first differences of the log prices is not uncorrelated.



Heston Model
Model & Assumptions

- Volatility of the risky asset is not constant but stochastic.
- Dynamics in the model:

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S$$
$$d\nu_t = \kappa(\theta - \nu_t)dt + \xi \sqrt{\nu_t} dW_t^\nu$$

where $\nu_t =$ *volatility process*
$\theta =$ *long run average price variance*
$\xi =$ *volatility of the volatility*

- Sources of uncertainty are correlated:

$$dW_t^S dW_t^\nu = \rho dt$$

Heston Model

Short-comings

- Hard to find proper parameters to calibrate the model.
- Fitness of the model depends on these parameters.
- Jumps in prices are not taken into account.

Delta-hedge for a European Call

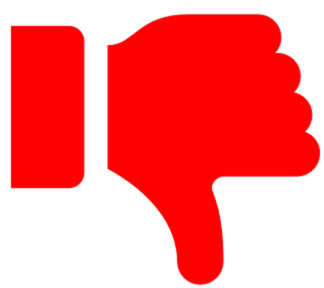
Setting

- European Call Option given an index.
- Strategy:
  - → Short in 1 call.
  - → Calculate the Δ of the option.
  - → Long Δ many future contracts on the index.
- Δ of this hedging strategy is 0, i.e. the portfolio is not affected by small changes in the underlying.

Delta-hedge for a European Call

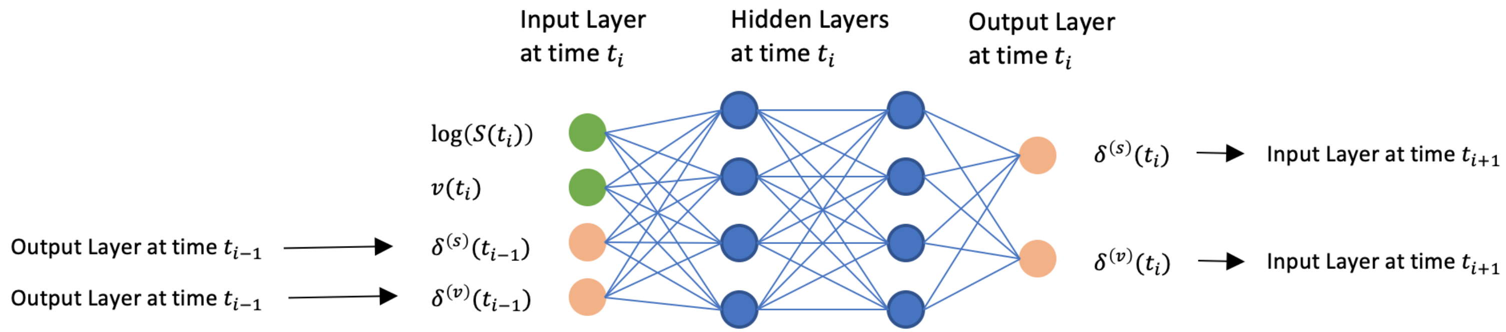
Limitations

- Assumption of complete markets with no trading costs.
- Δ based on specific model may be inappropriate for real market.
- Jumps in the underlying after realizing the delta strategy may be a problem.



Deep Reinforcement Learning

A different approach



- We want to hedge some portfolio z.
- We can input any relevant information which is available until time t.
- Our network has a LSTM structure inspired by rebalancing.
- The output will be the changes to the hedging portfolio.

Deep Reinforcement Learning

A different approach

- The goal of the strategy which is obtained by the network is given by:

$$\sup_{\pi \in \Lambda} v_t^{\pi}(z|s) = \sup_{\pi \in \Lambda} -C_t^{\pi}(s) + R\left[G_t^{\pi} | s_t = s\right]$$

  - → z is our portfolio which has to be hedged.
  - → C_t^{π} are costs at time t (e.g. fixed fee, or proportional).
  - → $R[\cdot]$ a risk measure.
  - → G_t^{π} are the future cash flows.
  - → s_t the current state of the economy.

- In our case, the authors chose CVaR(α): $R[X] = \sup_{w \in \mathbb{R}} \left(w - \frac{1}{1-\alpha} \mathbb{E}\left[(w - X)^+\right] \right)$.

Delta-hedge for a European Call
Advantages of neural networks

- The hedging strategy can be dependent on historical data but also simulated by theoretical models.
- It is easier to implement constraints (e.g. on capital, risk or regulations).
- The approach can consider market frictions (e.g. transaction costs).
- The model is computationally scaleable.
- OTC products can be priced.
- **The calculation of the hedging strategy is encapsulated and we can focus on modelling the market under realistic conditions.**

Training Algorithms
Introduction

The training of the model is done via the "Adam” optimizer in TensorFlow. Some features of "Adam" includes

- Gradient desend: The next value is calculated based on the previous value adjusted with the gradient.
- Adaptive learning rate: The bigger spread/uncertainty, the smaller the learning rate.
- Better updates to the parameters provides faster training.

Empirical Study

Problem setting

- Setup:
  - → We are the seller of one European call with strike K and time to maturity T (in days) with underlying price $(\cdots, S_{-1}, S_0)$.
  - → Hedging Strategy: A portfolio $\delta = (\delta_0, \delta_1, \ldots, \delta_{T-1})$ where δ_i is the number of asset we hold at time t. (Note at T the call is settled.)
  - → PnL:

$$\text{PnL} = -C + \sum_{i=0}^{n-1} \delta_i (S_{i+1} - S_i)$$

  - → Other assumptions can be easily added.
- Goal: We want to find a strategy that the combined portfolio has the lowest risk, evaluated by some risk measure $R(\text{PnL})$.
- Training data: The training dataset (market data) is generated by a discretized Black-Scholes series.

Empirical Study

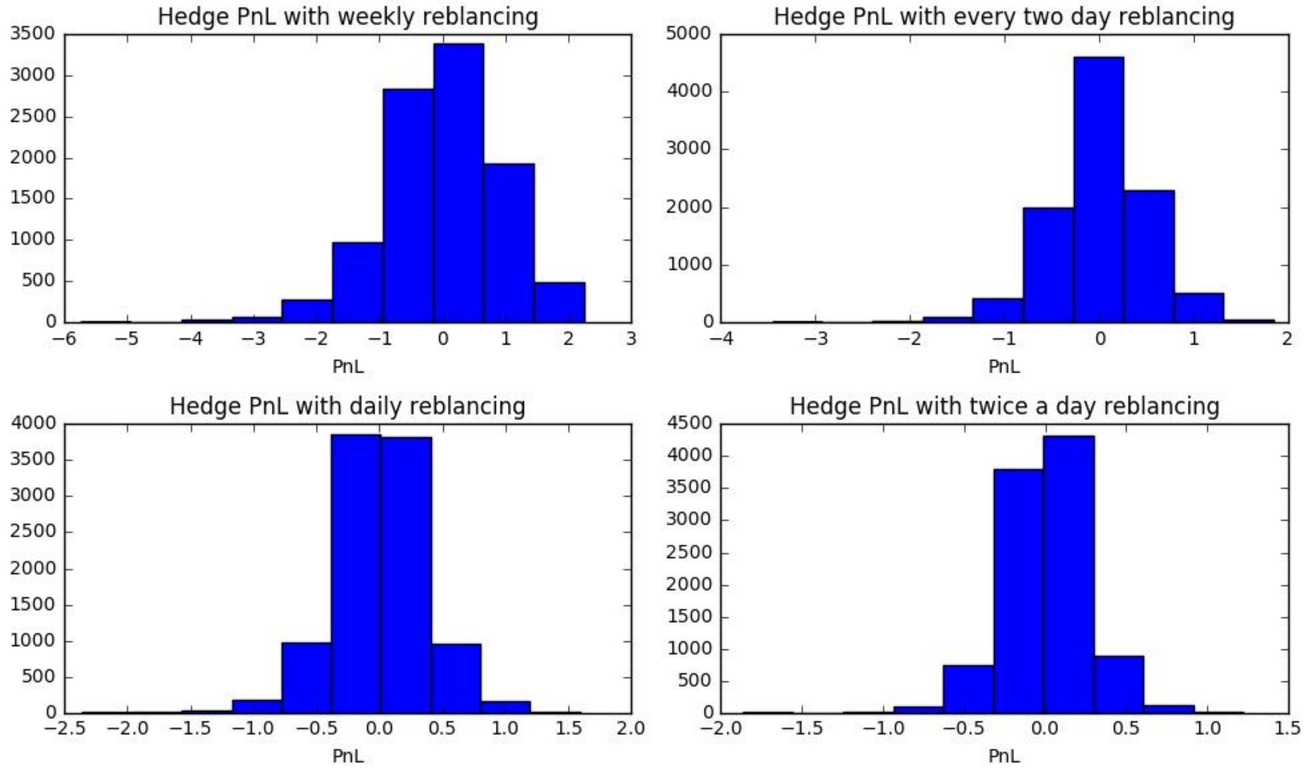
Benchmark strategy versus Deep Hedging

We simulate market data, train our RNN model, and then compare the out-of-sample model performance of deep hedging (portfolio given by the trained model) with the traditional B-S hedging (our benchmark model) in the following 3 scenarios:

- ◇ No transaction cost; model trained with B-S market prices; tested with B-S market prices.
- ◇ No transaction cost; model trained with B-S market prices; tested with Heston prices.
- ◇ With a percentage transaction cost; model trained with B-S market prices; tested with B-S prices.

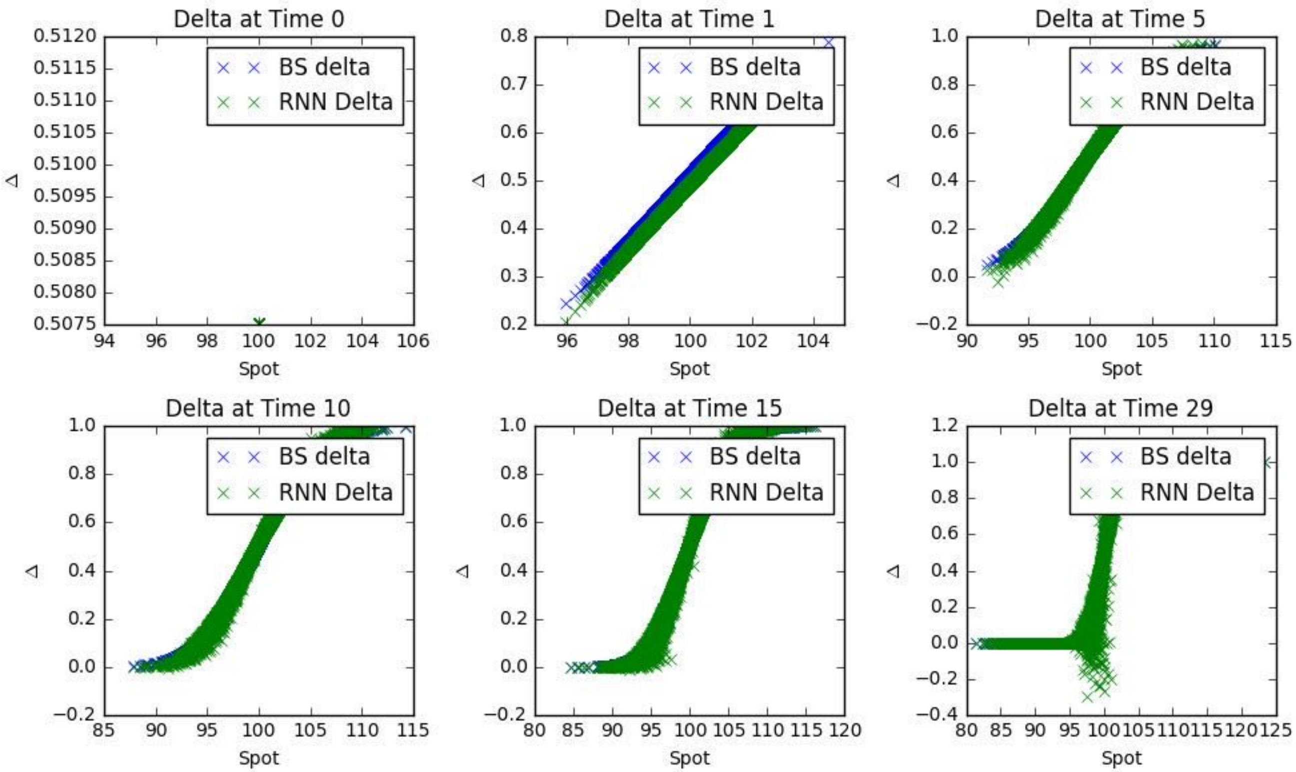
Empirical Study

Comparison of rebalancing periods



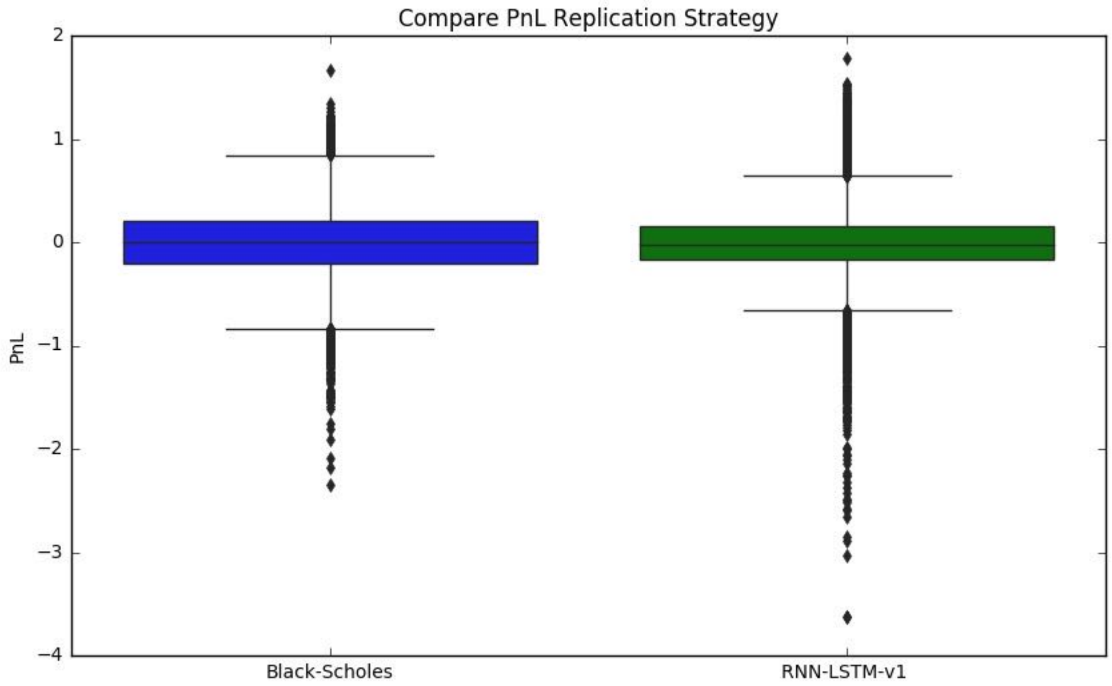
Empirical Study

Senario 1



Empirical Study

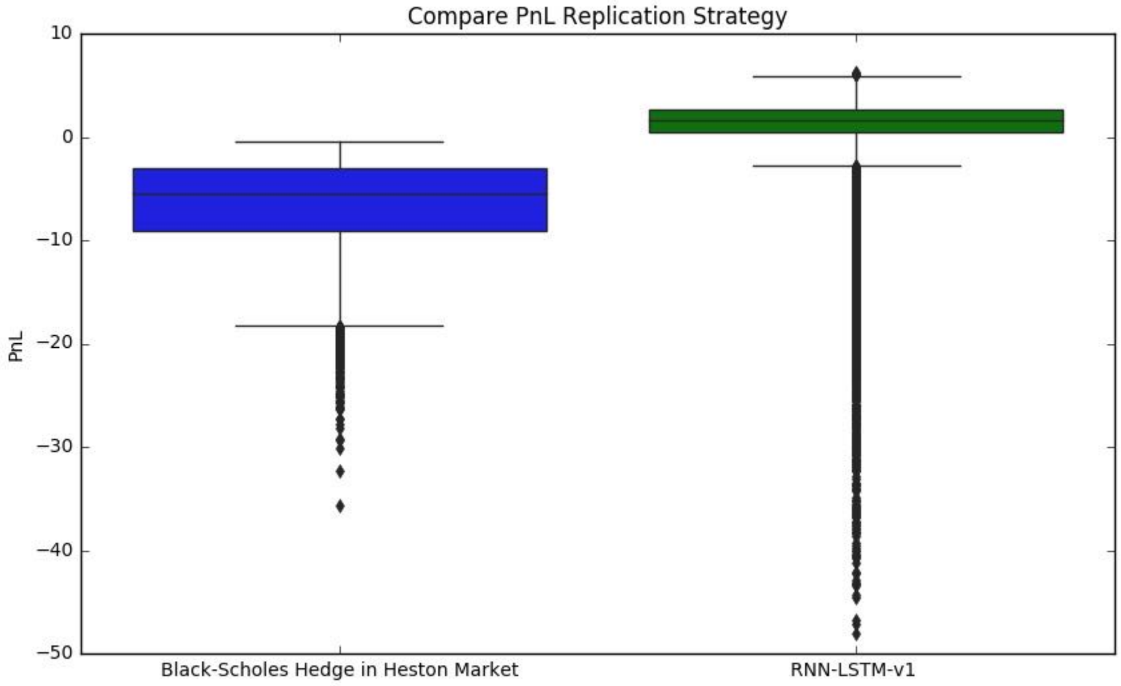
Senario 1



Average PnL: -0.001
99% CVaR: 1.225

Average PnL: 0.001
99% CVaR: 1.012

Empirical Study
Senario 2



Average PnL: -6.65
99% CVaR: 23.15

Average PnL: -0.12
99% CVaR: 35.28

Summary
Advantages of deep hedging over B-S delta hedging

- Robustness: model-free approach.
- Extensibility: market frictions, more assets, other constraints, and other risk measures.
- Scalability: computationally easier compared to traditional statistical approaches.

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Sources
and References

- Buehler, Hans and Gonon, Lukas and Teichmann, Josef and Wood, Ben and Mohan, Baranidharan and Kochems, Jonathan, Deep Hedging: Hedging Derivatives Under Generic Market Frictions Using Reinforcement Learning (March 19, 2019). *Swiss Finance Institute Research Paper No. 19-80*, Available at SSRN: *https://ssrn.com/abstract=3355706*